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Duality of (0,2) String Vacua

Jacques Distler^{*}

Theory Group

Department of Physics

University of Texas

Austin, TX 78712

and

Shamit Kachru[†]

Lyman Laboratory of Physics

Harvard University

Cambridge, MA 02138

We discuss a duality of (0,2) heterotic string vacua which implies that certain pairs of (0,2) Calabi-Yau compactifications on topologically distinct target manifolds yield identical string theories. Some complex structure moduli in one model are interpreted in the dual model as deforming the holomorphic structure of the vacuum gauge bundle (and *vice versa*). A better understanding of singularity resolution for (0,2) models may reveal that this duality of compactifications on singular spaces is part of a larger story, involving smooth topology-changing processes which interpolate between the (0,2) models on the resolved spaces.

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^{*}Email: `distler@utpapa.ph.utexas.edu` .

[†]Junior Fellow, Harvard Society of Fellows. Email: `kachru@string.harvard.edu` .

1. Introduction

(0,2) superconformal field theories with integral $U(1)$ charges and the appropriate central charge can be used to compactify the heterotic string to four dimensions, yielding string models with unbroken $N=1$ spacetime supersymmetry [1]. Unlike their (2,2) counterparts (which manifest an unbroken E_6 gauge symmetry), (0,2) theories can also yield models with an effective $SU(5)$ or $SO(10)$ unification group, and are therefore of particular interest for model building [2,3,4].

However, little is known about general (0,2) Calabi-Yau theories. In part, this is because the issue of conformal invariance has been clouded by worldsheet instantons [5]. Recent work [6,7] indicates that contrary to previous expectations, (0,2) Calabi-Yau σ -models are *not* destabilized by such nonperturbative sigma model effects.¹ Therefore, it is important to extend our detailed knowledge of (2,2) vacua to their (0,2) counterparts.

One of the most interesting “stringy” phenomena encountered in the exploration of (2,2) models is mirror symmetry [8,9,10] : The same physical theory can be obtained by considering string propagation on a manifold M or on its topologically distinct mirror \tilde{M} . This symmetry (like other examples of duality in string theory) is completely unexpected from the perspective of a Kaluza-Klein theorist.

One might wonder what other sorts of duality occur in more general (0,2) models. In this paper we explore a duality first noted in [11]. A (0,2) Calabi-Yau model requires for its specification both a manifold M and a stable, holomorphic vector bundle $V \rightarrow M$, the vacuum gauge bundle. The duality we discuss relates a (0,2) model with data (M, V) to a (0,2) model with data (\tilde{M}, \tilde{V}) , with M and \tilde{M} topologically distinct. As in mirror symmetry, the dual pairs should provide different descriptions of the *same* physical theory.

An interesting possibility, also mentioned in [11], is that this duality is a small part of a larger story. We will find numerous examples of dual pairs, but in each of our examples singularities on M or \tilde{M} , which are “frozen in” on the part of the moduli space we can study, will complicate the analysis. It could be that when the resolution of these singularities is properly understood, one will find phenomena analogous to those of [12,13]. Namely, it could be that there will be additional moduli which resolve the singularities, and that by varying these moduli we can pass through a “wall” separating topologically distinct “phases” – corresponding to the smooth (M, V) and (\tilde{M}, \tilde{V}) theories. If this is the case,

¹ The conclusions of the forthcoming paper [7] in particular are stronger than those of [6] and imply that all of the models considered here are *bona fide* solutions of string theory.

the duality we see here is merely the statement that on this wall – in the theory on the singular manifolds – one can ascribe either geometrical interpretation to the conformal theory.

The moduli of a (0,2) Calabi-Yau model come in three types: Elements of $H^{1,1}(M)$ and $H^{2,1}(M)$ which deform the Kähler and complex structure of the base manifold M , and elements of $H^1(M, \text{End}(V))$ which correspond to deformations of the holomorphic structure of the gauge bundle V . The duality we discuss exchanges some elements of $H^{2,1}(M)$ with elements of $H^1(\tilde{M}, \text{End}(\tilde{V}))$.

In §2 we review the phases picture of the moduli spaces of (0,2) Calabi-Yau models [12,11], and provide evidence for our (0,2) duality by finding examples of distinct (0,2) Calabi-Yau models with isomorphic Landau-Ginzburg phases. This extends in a more or less straightforward manner to an argument that the models remain isomorphic on the full subspace of their moduli spaces described by our $U(1)$ gauged linear sigma models. §3 contains a large radius check of the equivalence of pairs related by the phenomenon of §2. We make the easy observation that the unnormalized Yukawa couplings of the generations in the dual descriptions are equal, and also make some remarks about the dependence of the Yukawa couplings in such (0,2) models on the Kähler modulus. Some open questions are discussed in §4.

2. Phase Diagrams and Dual Pairs

A useful tool in understanding the moduli spaces of many Calabi-Yau models is the gauged linear sigma model, introduced in this context by Witten [12] (an equivalent description, in the language of toric geometry, is given in [13]). The basic idea is to study spaces of gauged linear sigma models which, in the infrared limit, approach the conformally invariant theories of interest. Following [11] (and in particular using the same notation and (0,2) multiplets explained in that paper), we now show the existence of dual pairs of (0,2) vacua by studying appropriate gauged (0,2) linear sigma models.

2.1. A Class of (0,2) Calabi-Yau Models

The data involved in specifying the (0,2) theories of interest to us includes a Calabi-Yau manifold M and a stable, holomorphic vector bundle $V \rightarrow M$ satisfying

$$c_2(V) = c_2(TM), \quad c_1(V) = 0 \bmod 2. \quad (2.1)$$

The first condition is the familiar anomaly cancellation condition, while the second condition guarantees that the bundle V admits spinors.

Beyond these topological conditions, there are of course perturbative requirements for conformal invariance of a (0,2) Calabi-Yau sigma model [2]. At lowest order, we must choose the Kähler metric $g_{i\bar{j}}$ on M to be the Ricci-flat metric whose existence is guaranteed by Yau's theorem, and in addition we must choose the connection on V to satisfy the Donaldson-Uhlenbeck-Yau equation

$$g^{i\bar{j}} F_{i\bar{j}} = 0. \quad (2.2)$$

The integrability condition for existence of a solution to (2.2) is

$$\int_M J \wedge J \wedge c_1(V) = 0 \quad (2.3)$$

where J is the Kähler form of M [14]. We will meet this condition by requiring $c_1(V) = 0$. Higher orders of sigma model perturbation theory do not lead to any new conditions on M or $V_{1,2}$.

We will concentrate on complete intersection Calabi-Yau manifolds defined by the intersection of N hypersurfaces

$$W_i(\phi) = 0, \quad i = 1, \dots, N \quad (2.4)$$

of degree d_i in some $WCP_{w_1, \dots, w_{4+N}}^{3+N}$ with homogeneous coordinates $\phi_1, \dots, \phi_{4+N}$. We use bundles V defined as the kernel of an exact sequence

$$0 \rightarrow V \rightarrow \bigoplus_{a=1}^{\tilde{r}+1} \mathcal{O}(n_a) \xrightarrow{\otimes F_a(\phi)} \mathcal{O}(m) \rightarrow 0 \quad (2.5)$$

where m and n_a are positive integers, $\mathcal{O}(j)$ denotes the j th power of the hyperplane bundle of the ambient weighted projective space, and the $F_a(\phi)$ are polynomials homogeneous of degree $m - n_a$ in the ϕ s. The constraints (2.1) are translated to conditions on the integers m , n_a , and d_i :

$$m^2 - \sum_{a=1}^{\tilde{r}+1} n_a^2 = \sum_{i=1}^N d_i^2 - \sum_{j=1}^{4+N} w_j^2, \quad m = \sum_{a=1}^{\tilde{r}+1} n_a. \quad (2.6)$$

In addition, one of course can write the Calabi-Yau condition as

$$\sum_{i=1}^N d_i = \sum_{j=1}^{N+4} w_j. \quad (2.7)$$

2.2. A CY Manifold as a Gauged Linear Sigma Model

Let us now rephrase §2.1 in the language of quantum field theory. As an example, let us start by discussing the gauged linear sigma model description of the complete intersection M of three hypersurfaces $W_i(\phi) = 0$ of degrees 3, 6, 6 in $WCP_{1,1,1,1,3,3,5}^6$ with gauge bundle V specified by $m = 7$ and $\{n_a\} = \{1, 1, 2, 3\}$. Introduce a worldsheet $U(1)$ gauge field, seven chiral superfields Φ_j , four left-moving fermi multiplets Λ_a , three left-moving fermi multiplets Σ_i , and one additional chiral superfield P . Assign the Φ_j gauge charges $w_j = 1, 1, 1, 1, 3, 3, 5$, the Λ_a charges $n_a = 1, 1, 2, 3$, the Σ_i charges $d_i = -3, -6, -6$, and P charge $m = -7$. In addition to the normal kinetic terms for all of these multiplets, introduce a $U(1)$ Fayet-Iliopoulos D-term with coefficient r and a (0,2) superpotential

$$\mathcal{W} = \int d^2z d\theta \Sigma_i W_i(\phi) + P \Lambda_a F_a(\phi) . \quad (2.8)$$

Integrating out the D auxiliary field in the gauge multiplet and the auxiliary fields in the Fermi multiplets, we find the scalar potential

$$U = \sum_{i=1}^3 |W_i(\phi)|^2 + |p|^2 \sum_{a=1}^4 |F_a(\phi)|^2 + \frac{e^2}{2} \left(\sum_{j=1}^{M+4} w_j |\phi_j|^2 - m |p|^2 - r \right)^2 . \quad (2.9)$$

Now, we study the infrared behavior of this theory by focusing on the locus of vanishing U . While in general this is not necessarily a good approximation to the infrared physics, for $|r|$ very large studying only the ground states and the massless excitations around them does suffice. Integrating out the massive fields, for large $|r|$, simply induces corrections to the parameters of the low energy theory.

For r very large and positive, we see that the minimum of (2.9) is obtained when

$$p = 0, \quad \sum_j w_j |\phi_j|^2 = r, \quad W_i(\phi) = 0 . \quad (2.10)$$

Dividing the space of solutions of $\sum_j w_j |\phi_j|^2$ by the $U(1)$ gauge group precisely gives us $WCP_{1,1,1,1,3,3,5}^6$ with Kähler class proportional to r . The last constraint in (2.10) then tells us that the space of ground states for large r is precisely the Calabi-Yau manifold M .

What about the fermions? As in the (2,2) theories discussed in [12], the superpartners of the ϕ_i (which are right-moving fermions) transform as sections of the tangent bundle TM . The Yukawa coupling

$$\psi_p \lambda_a F_a(\phi) \quad (2.11)$$

with the fermionic partner of p gives a mass to one linear combination of the λ s, while the rest transform as sections of the rank \tilde{r} bundle V . Therefore, for large positive r we have recovered the (0,2) Calabi-Yau sigma model described by $V \rightarrow M$.

Next, we see what happens for r very negative, where semiclassical reasoning again should be a good guide to the infrared physics. Minimizing (2.9) we this time find

$$|p|^2 = r, \quad \phi_j = 0. \quad (2.12)$$

The gauge symmetry is broken to a Z_7 because p has gauge charge -7 ; and p and ψ_p become massive and drop out of the theory. What remains is then a (0,2) Landau-Ginzburg theory, with superpotential (after rescaling fields to absorb the VEV of p)

$$\int d^2z \, d^2\theta \, \Sigma_i W_i(\phi) + \Lambda_a F_a(\phi). \quad (2.13)$$

The Z_7 discrete gauge symmetry means that we wish to study not the theory (2.13), but its Z_7 orbifold. So by studying the “phases” of the (0,2) gauged LSM, we have been able to recover the Calabi-Yau/Landau-Ginzburg correspondence [15,16].

Note one interesting fact about the superpotential (2.13) (and indeed the full effective Lagrangian in the Landau-Ginzburg phase) – the model has “forgotten” about the distinction between the defining equations of the manifold W_i and the data F_a defining the holomorphic structure of the gauge bundle. While they enter in the Lagrangian in different ways at large radius, here at small radius they are on an equal footing.

2.3. Another CY Manifold as a Gauged Linear Sigma Model

Now, let's repeat the story of §2.2, this time on the manifold \tilde{M} defined by the intersection of degree 4, 5, 6 hypersurfaces

$$\tilde{W}_i(\phi) = 0 \quad (2.14)$$

again in $WCP^6_{1,1,1,1,3,3,5}$. This time, we choose \tilde{V} to be defined by $m = 7$ and $\{n_a\} = \{1, 1, 1, 4\}$.

At large positive r we recover once again the (0,2) CY sigma model with target \tilde{M} , with the right-moving fermions transforming as sections of $T\tilde{M}$ and the left-movers as sections of \tilde{V} . This sigma model is *a priori* unrelated to the one described in §2.2 – in

fact, from the viewpoint of classical geometry the pairs (M, V) and (\tilde{M}, \tilde{V}) are manifestly distinct. For example, M and \tilde{M} have distinct orbifold Euler characters

$$\chi(M) = -216, \quad \chi(\tilde{M}) = -192 \quad (2.15)$$

(in calculating (2.15) one has to resolve a Z_5 fixed point on M and two Z_3 fixed points on \tilde{M}).

Let's proceed to take the gauged linear sigma model description of the theory with target \tilde{M} and vacuum gauge bundle \tilde{V} down to $r \rightarrow -\infty$. Having introduced gauge charges and such as in §2.2, we see that at $r \rightarrow -\infty$ we are left with a Landau-Ginzburg theory possessing a discrete Z_7 gauge symmetry with superpotential

$$\int d^2z \, d^2\theta \, \tilde{\Sigma}_i \tilde{W}_i(\phi) + \tilde{\Lambda}_a \tilde{F}_a(\phi) . \quad (2.16)$$

Now we notice something interesting. The polynomials $\tilde{W}_i(\phi)$ have degrees 4, 5, 6 and the polynomials $\tilde{F}_a(\phi)$ have degrees 6, 6, 6, 3. Looking back to §2.2, in the Landau-Ginzburg phase of (M, V) the polynomials $W_i(\phi)$ have degrees 3, 6, 6 while the polynomials $F_a(\phi)$ have degrees 6, 6, 5, 4.

Now suppose we choose the data defining the two models as follows:

$$W_1(\phi) = \tilde{F}_4(\phi), \quad W_2(\phi) = \tilde{F}_3(\phi), \quad W_3(\phi) = \tilde{W}_3(\phi) \quad (2.17)$$

$$F_1(\phi) = \tilde{F}_1(\phi), \quad F_2(\phi) = \tilde{F}_2(\phi), \quad F_3(\phi) = \tilde{W}_2(\phi), \quad F_4 = \tilde{W}_1(\phi) . \quad (2.18)$$

Then the full Lagrangians defining the Landau-Ginzburg “phases” of the two models are identical, with the simple change of notation

$$(\Sigma_1, \Sigma_2, \Sigma_3, \Lambda_1, \Lambda_2, \Lambda_3, \Lambda_4) \rightarrow (\tilde{\Lambda}_4, \tilde{\Lambda}_3, \tilde{\Sigma}_3, \tilde{\Lambda}_1, \tilde{\Lambda}_2, \tilde{\Sigma}_2, \tilde{\Sigma}_1) . \quad (2.19)$$

So at the Landau-Ginzburg radius there is a duality which exchanges the two models, with the “duality map” given in (2.17), (2.18), (2.19).

This proves that the two naively distinct Calabi-Yau sigma models become isomorphic in their Landau-Ginzburg phases. Furthermore, the Kähler modulus r representing the overall size of the ambient WCP^6 arises as a twist field in the Landau-Ginzburg theory. Perturbation theory in this twist field will be identical for the two theories, so the identity between them extends off the Landau-Ginzburg locus to an open set in their Kähler moduli spaces (the region of convergence of the perturbation expansion in the twist field). But two

models which agree in such a region in the (complexified) Kähler moduli space spanned by r and the worldsheet θ angle must agree on the whole (r, θ) plane. So we find that the two models remain isomorphic for all values of r (and θ).

It is important to note that in the discussion of the Kähler moduli space above, we really mean only the Kähler modulus corresponding to the parameters (r, θ) in the linear sigma model. In the (2,2) models on these manifolds, the full Kähler moduli spaces would be three (complex) dimensional, with two additional elements of $h_{1,1}$ being introduced by resolving the Z_5 fixed point on M and the two Z_3 fixed points on \tilde{M} . In these (0,2) models, the status of these additional elements of $h_{1,1}$ is less clear. It could be that once these moduli, and perhaps other additional moduli which are related to the resolution of the singularities of (M, V) and (\tilde{M}, \tilde{V}) , are added, the duality of this subspace of the moduli spaces of (M, V) and (\tilde{M}, \tilde{V}) will be seen as part of a larger story involving topology change as in [12,13].

We should also mention now that the isomorphism (2.17)(2.18)(2.19) manifestly mixes the W_i and Σ_i with the \tilde{F}_a and the $\tilde{\Lambda}_a$. But perturbations of the \tilde{F}_a correspond, at large radius, to changes in the holomorphic structure of \tilde{V} , while perturbations of the W_i correspond to changes in the complex structure of M . So we see that the duality we are studying involves the exchange of elements of $H^{2,1}(M)$ with elements of $H^1(\tilde{M}, \text{End}(\tilde{V}))$.

2.4. More Examples

In §2.3 we have seen the game that one has to play in order to find (0,2) models which will be dual pairs. Take two complete intersection manifolds in the same weighted projective space and choose vacuum gauge bundles over each with the same m . Then as long as the full sets of degrees of the W s and the F s in one model and the \tilde{W} s and \tilde{F} s in the other model coincide, the two theories will be isomorphic in their Landau-Ginzburg phases. So by searching for different complete intersections and bundles in the same projective space which satisfy these criteria, we can find dual pairs.

In Table 1 below, we list several examples. Under M and \tilde{M} we list the degrees of the hypersurfaces defining the complete intersection manifolds, under V and \tilde{V} we give the data $(m; \{n_a\})$, and under $\{w_j\}$ we list the weights of the ambient weighted projective space. In calculating the Euler characters of the singular manifolds below, we have used the orbifold Euler characteristic formula of the second reference in [16]. The first example is the one discussed in §2.2-2.3 and the last example was given in [11].

M	V	\tilde{M}	\tilde{V}	$\chi(M)$	$\chi(\tilde{M})$	$\{w_j\}$
3, 6, 6	(7; 1, 1, 2, 3)	4, 5, 6	(7; 1, 1, 1, 4)	-216	-192	1, 1, 1, 1, 3, 3, 5
3, 11	(12; 1, 1, 3, 7)	5, 9	(12; 1, 1, 1, 9)	-280	-216	1, 1, 1, 3, 3, 5
4, 4, 4	(6; 1, 1, 1, 3)	3, 4, 5	(6; 1, 1, 2, 2)	-120	-132	1, 1, 1, 2, 2, 2, 3
3, 4, 7	(8; 1, 1, 2, 4)	4, 4, 6	(8; 1, 1, 1, 5)	-132	-112	1, 1, 2, 2, 2, 3, 3
6, 6	(8; 1, 1, 3, 3)	5, 7	(8; 1, 2, 2, 3)	-120	-120	1, 1, 2, 2, 3, 3
3, 8	(9; 1, 1, 2, 5)	4, 7	(9; 1, 1, 1, 6)	-168	-132	1, 1, 2, 2, 3
5, 8	(10; 1, 1, 2, 6)	4, 9	(10; 1, 2, 2, 5)	-104	-108	1, 2, 2, 2, 3, 3

Table 1: Some examples of dual pairs of (0,2) Calabi-Yau σ -models on Calabi-Yau threefolds.

Each of these pairs, when represented as gauged linear sigma models with a single $U(1)$ gauge group, has the feature that they are manifestly isomorphic in their Landau-Ginzburg phases. By the perturbative argument given in §2.3, we expect these theories to remain isomorphic at finite (r, θ) as well. We have no insights about the possibility of resolving these models while preserving the conditions (2.1). Note however that the manifold M in the fifth dual pair listed in Table 1 is already nonsingular, while its dual is not.

3. A Large Radius Test: Yukawa Couplings

We have given general arguments that the pairs of sigma models on topologically distinct Calabi-Yau manifolds listed in Table 1 give isomorphic superstring vacua. Still, this seems fairly mysterious at large radius. In order to convince the reader that it is nonetheless true, we now discuss the large radius Yukawa couplings of the first dual pair listed in Table 1. Everything we say is dependent on the (very plausible) assumption that the point singularities of M , \tilde{M} do not affect our considerations, and generalizes in an obvious way to the other examples of §2.4.

These models both have an observable E_6 gauge group in spacetime. Choose the defining data to obey the constraints (2.17)(2.18). Computing the spectrum of the model at the Landau-Ginzburg point following [17,11], we find 123 $\mathbf{27}$ s of E_6 and 1 $\overline{\mathbf{27}}$ of E_6 . 122 of the $\mathbf{27}$ s come from the untwisted sector and have a deformation-theoretic representation as seventh degree polynomials

$$\{P_7(\phi)\} \bmod \{W_i(\phi), F_a(\phi)\} . \quad (3.1)$$

This is the expected large radius answer for both models. **27**s of E_6 should be in 1-1 correspondence with elements of $H^1(M, V)$ and $H^1(\tilde{M}, \tilde{V})$ in the two models, and it follows from the sequence (2.5) that $H^1(M, V)$ and $H^1(\tilde{M}, \tilde{V})$ should have such a deformation theoretic representation.

What about the extra **27** and $\overline{\mathbf{27}}$ in the Landau-Ginzburg theory? They both come from twisted sectors. There are two possibilities – either they survive at large radius and can be understood as coming from the resolution of the singularities, or they pair up as one leaves the Landau-Ginzburg point (the latter possibility is consistent with all of the symmetries of the Landau-Ginzburg theory). We shall not try to distinguish between these two possibilities here, though it certainly is possible to study the relevant correlation functions in the Landau-Ginzburg theory. Instead, we shall focus henceforth on the 122 deformation theoretic **27**s in each model, whose origin we easily understand both at small and large radius.

At large radius, the Yukawa couplings are given by the intersection forms

$$H^1(M, V) \otimes H^1(M, V) \otimes H^1(M, V) \rightarrow \mathbb{C} \quad (3.2)$$

$$H^1(\tilde{M}, \tilde{V}) \otimes H^1(\tilde{M}, \tilde{V}) \otimes H^1(\tilde{M}, \tilde{V}) \rightarrow \mathbb{C} \quad (3.3)$$

where we use $H^3(M, \bigwedge^3 V) = H^3(M, \mathcal{O}) = H^{0,3}(M) = \mathbb{C}$ and $H^3(\tilde{M}, \bigwedge^3 \tilde{V}) = H^{0,3}(\tilde{M}) = \mathbb{C}$. This follows from the fact that $c_1(V) = c_1(\tilde{V}) = 0$, so that for V, \tilde{V} of rank 3, $\bigwedge^3 V$ and $\bigwedge^3 \tilde{V}$ are trivial. Following the discussion in [18] and especially §4 and §5 of [19], it is easy to see what the relevant formula for the unnormalized Yukawa couplings will be.

Consider the coupling κ_{ijk} of three generations represented by cohomology classes A_i^a , A_j^b and A_k^c where $A_i^a = A_{\bar{m}i}^a d\bar{z}^m \in H_{\bar{\partial}}^{(0,1)}(M, V)$. This has an expression as an integral over the manifold

$$\kappa_{ijk} = \int_M \Omega \wedge A_i^a \wedge A_j^b \wedge A_k^c \epsilon_{abc} \quad (3.4)$$

where Ω is the holomorphic three-form of M . Now, define

$$\bar{\omega} = A_i^a \wedge A_j^b \wedge A_k^c \epsilon_{abc} . \quad (3.5)$$

It follows from (3.5) that $\bar{\omega}$ is a closed (0,3) form. But on a Calabi-Yau manifold $\bar{\Omega}$ is the unique element of $H^{0,3}(M)$, so we may decompose

$$\bar{\omega} = \kappa \bar{\Omega} + \bar{\partial}(\cdots) \quad (3.6)$$

and since the exact part will not contribute in the integral (3.4), the constant κ in (3.6) is equal to the Yukawa coupling.

For the models of interest the elements $A_i^a \in H_{\bar{\partial}}^{(0,1)}(M, V)$ have a deformation theoretic representation (3.1). Similarly, the antiholomorphic (0,3) form $\bar{\Omega}$ can be represented (up to scale) by the single 21^{st} degree polynomial not in the ideal generated by the W_i and the F_a . Call the single element in the quotient \hat{P}_{21}

$$\hat{P}_{21} \in \{ \{P_{21}(\phi)\} \bmod \{W_i(\phi), F_a(\phi)\} \} . \quad (3.7)$$

The rule for computing the (unnormalized) Yukawa couplings follows in a straightforward manner – if A_i^a , A_j^b and A_k^c have polynomial representatives $P_{i,j,k}$, then the coupling κ_{ijk} follows from

$$P_i P_j P_k = \kappa_{ijk} \hat{P}_{21} + \dots . \quad (3.8)$$

Now it is clear that the unnormalized Yukawa couplings of the deformation theoretic generations for all of the dual pairs listed in §2.4 will agree. The point is that the polynomial representatives for the generations (3.1) and the three-form (3.7) which determine the Yukawa couplings via (3.8) can be chosen to be *identical* for the dual pairs precisely when the conditions analogous to (2.17)(2.18) are satisfied.

It worth emphasizing that the Yukawa couplings of the deformation theoretic generations can also be considered directly at the Landau-Ginzburg radius [20]. The results are in fact exactly the same as the large radius results we have just described. This should be taken as strong evidence that the Yukawa couplings of the deformation theoretic generations are in fact *independent* of r – like the couplings between (2,1) forms in (2,2) Calabi-Yau models [21].

4. Conclusion

In this paper we have described a new kind of duality of Calabi-Yau models. Mirror symmetry exchanges deformations of the size of one manifold with deformations of the shape of its mirror. The duality described here exchanges “gravitational” moduli of one classical solution (deformations of the shape of the compactification manifold) with “gauge” moduli of its dual (deformations of the gauge field VEV). Like other dualities of string vacua [22], this is certainly not something our particle theory intuition would lead us to expect.

There are many aspects of the (0,2) duality discussed here that we do not understand, however. Even in the examples presented, there are mysteries. This was clear in §3 – our large radius understanding of the spectrum and interactions of charged particles is incomplete, partially because of difficulties with singularities. The questions we need to answer are:

- 1) When can singular (0,2) models be resolved maintaining (2.1) ?
- 2) What effect do the added moduli have on our picture of the moduli spaces?

It could well be that once singularity resolution is understood, this duality will be seen as a phenomenon on the “wall” of smooth topology changing processes connecting (M, V) to (\tilde{M}, \tilde{V}) .

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References

- [1] T. Banks, L. Dixon, D. Friedan, and E. Martinec, “Phenomenology and Conformal Field Theory or Can String Theory Predict the Weak Mixing Angle?”, *Nucl. Phys.* **B299** (1988) 613.
- [2] E. Witten, “New Issues in Manifolds of SU(3) Holonomy,” *Nucl. Phys.* **B268** (1986) 79.
- [3] J. Distler and B. Greene, “Aspects of (2,0) String Compactifications,” *Nucl. Phys.* **B304** (1988) 1.
- [4] S. Kachru, “Some Three Generation (0,2) Calabi-Yau Models,” Harvard preprint to appear.
- [5] M. Dine, N. Seiberg, X. Wen, and E. Witten, “Non-Perturbative Effects on the String World Sheet I,II,” *Nucl. Phys.* **B278** (1986) 769, **B289** (1987) 319.
- [6] J. Distler and S. Kachru, “Singlet Couplings and (0,2) Models,” *Nucl. Phys.* **B430** (1994) 13, [hep-th/9406090](#).
- [7] E. Silverstein and E. Witten, to appear.
- [8] B. Greene and M. Plesser, “Duality in Calabi-Yau Moduli Space,” *Nucl. Phys.* **B338** (1990) 15.
- [9] P. Candelas, X. de la Ossa, P. Green, and L. Parkes, “A Pair of Calabi-Yau Manifolds as an Exactly Soluble Superconformal Theory,” *Nucl. Phys.* **B359** (1991) 21.
- [10] S.T. Yau, ed., *Essays on Mirror Manifolds*, International Press, 1991.
- [11] J. Distler and S. Kachru, “(0,2) Landau-Ginzburg Theory,” *Nucl. Phys.* **B413** (1994) 213, [hep-th/9309110](#).
- [12] E. Witten, “Phases of N=2 Theories in Two Dimensions,” *Nucl. Phys.* **B403** (1993) 159, [hep-th/9301042](#).
- [13] P. Aspinwall, B. Greene, and D. Morrison, “Calabi-Yau Moduli Space, Mirror Manifolds, and Spacetime Topology Change in String Theory,” *Nucl. Phys.* **B416** (1994) 414, [hep-th/9309097](#).
- [14] K. Uhlenbeck and S.T. Yau, “On the Existence of Hermitian-Yang-Mills Connections in Stable Vector Bundles,” *Comm. Pure. App. Math.* Vol. XXXIX (1986) S257.
- [15] E. Martinec, “Criticality, Catastrophes, and Compactifications,” in *Physics and Mathematics of Strings*, ed. L. Brink, D. Friedan, and A.M. Polyakov, World Scientific, 1992.
- [16] C. Vafa and N. Warner, “Catastrophes and the Classification of Conformal Field Theories,” *Phys. Lett.* **218B** (1989) 51; B. Greene, C. Vafa, and N. Warner, “Calabi-Yau Manifolds and Renormalization Group Flows,” *Nucl. Phys.* **B324** (1989) 371.
- [17] S. Kachru and E. Witten, “Computing the Complete Massless Spectrum of a Landau-Ginzburg Orbifold,” *Nucl. Phys.* **B407** (1993) 637, [hep-th/9307038](#).

- [18] A. Strominger and E. Witten, “New Manifolds for Superstring Compactification,” *Comm. Math. Phys.* **101** (1985) 341.
- [19] P. Candelas, “Yukawa Couplings Between (2,1) Forms,” *Nucl. Phys.* **B298** (1988) 458.
- [20] T. Kawai and K. Mohri, “Geometry of (0,2) Landau-Ginzburg Orbifolds,” *Nucl. Phys.* **B425** (1994) 191, [hep-th/9402148](#).
- [21] J. Distler and B. Greene, “Some Exact Results on the Superpotential from Calabi-Yau Compactifications,” *Nucl. Phys.* **B309** (1988) 295.
- [22] A. Giveon, M. Porrati, and E. Rabinovici, “Target Space Duality in String Theory,” *Phys. Rept.* **244** (1994) 77, [hep-th/9401139](#).